basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P1

2018

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 11 questions.

2.   Answer ALL the questions.

3.   Number the answers correctly according to the numbering system used in this question paper.

4.   Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

5.   Answers only will NOT necessarily be awarded full marks.

6.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

8.   Diagrams are NOT necessarily drawn to scale.

9.   An information sheet with formulae is included at the end of the question paper.

10.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x:

    1.1.1 $(3x-1)(x+4)=0$ (2)

    1.1.2 $2x^2+9x-14=0$ (correct to TWO decimal places)   (4)

    1.1.3 $\sqrt{3-26x}=3x$ (4)

    1.1.4 $(x-1)(x-4)>x+11$ (5)

1.2   Simplify fully:

$$\frac{\sqrt{16x^7}-\sqrt{25x^7}}{\sqrt{x}}$$

    (3)

1.3   Solve simultaneously for x and y:

$xy=9$ and $x-2y-3=0$ (5)

1.4   Prove that $x^2+2xy+2y^2$ cannot be negative for $x,y\in\mathbb{R}$.   (4)

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QUESTION 2

2.1   Given the quadratic pattern: $5\,;\,10\,;\,17\,;\,26\,;\,\ldots$

    2.1.1   Write down the next TWO terms of the pattern.   (2)

    2.1.2   Determine the formula for the nth term of the pattern.   (4)

    2.1.3   Which term of the pattern will have a value of 1 765?   (4)

2.2   The first 24 terms of an arithmetic series are: $35+42+49+\ldots+196$.

Calculate the sum of ALL natural numbers from 35 to 196 that are NOT divisible by 7.   (5)

[15]

QUESTION 3

Themba is planning a bicycle trip from Cape Town to Pretoria. The total distance covered during the trip will be 1 500 km. He plans to travel 100 km on the first day. For every following day he plans to cover 94% of the distance he covered the previous day.

3.1   What distance will he cover on day 3 of the trip?   (2)

3.2   On what day of the trip will Themba pass the halfway point?   (4)

3.3   Themba must cover a certain percentage of the previous day's distance to ensure that he will eventually reach Pretoria. Calculate ALL possible value(s) of this percentage.   (3)

[9]

QUESTION 4

The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\dfrac{16}{9}; p\right)$ is a point on f.
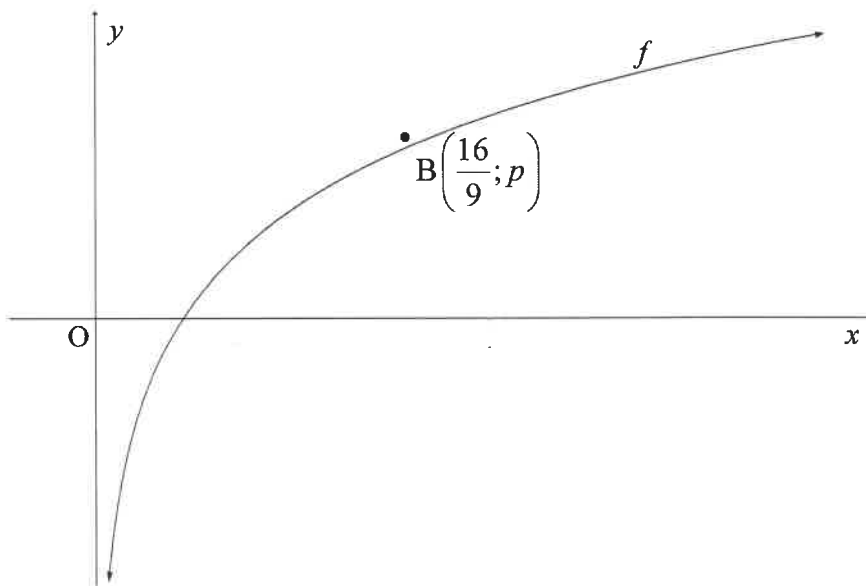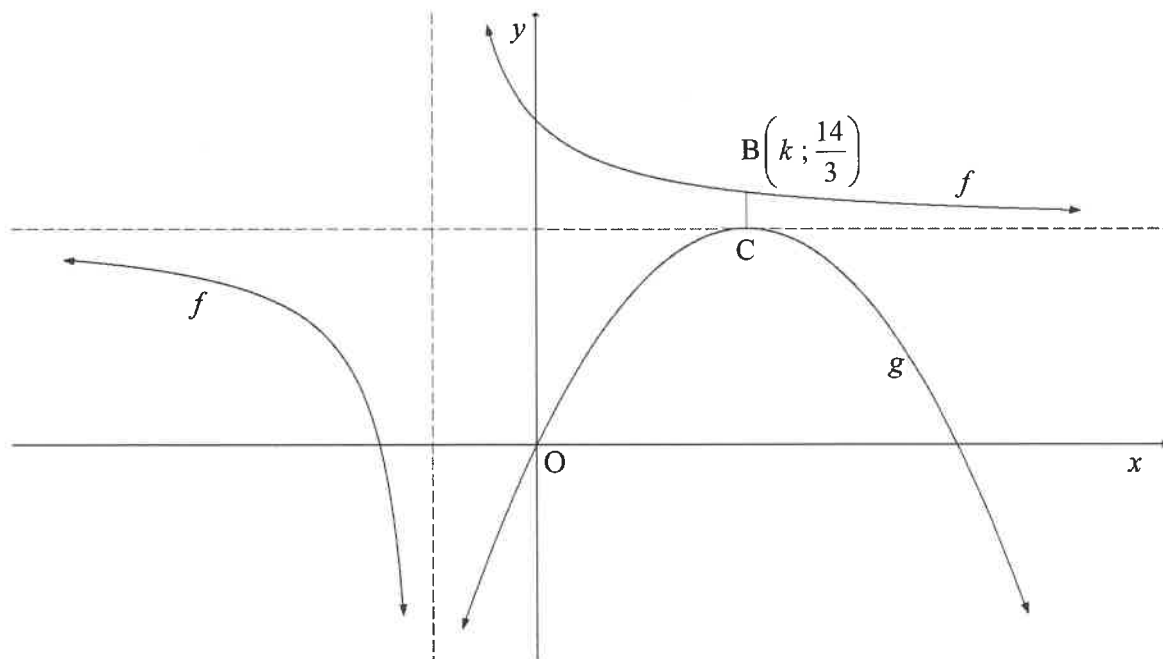

4.1   For which value(s) of x is $\log_{\frac{4}{3}} x \leq 0$?   (2)

4.2   Determine the value of p, without the use of a calculator.   (3)

4.3   Write down the equation of the inverse of f in the form $y = \ldots$ (2)

4.4   Write down the range of $y = f^{-1}(x)$.   (2)

4.5   The function $h(x) = \left(\dfrac{3}{4}\right)^x$ is obtained after applying two reflections on f.

Write down the coordinates of B'', the image of B on h.   (2)

[11]

QUESTION 5

The graphs of $f(x) = \dfrac{2}{x+1} + 4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptote of f.
- The graph of g passes through the origin.
- $B\left(k; \dfrac{14}{3}\right)$ is a point on f such that BC is parallel to the y-axis.

5.1	Write down the domain of f.	(2)
5.2	Determine the x-intercept of f.	(2)
5.3	Calculate the value of k.	(3)
5.4	Write down the coordinates of C.	(2)
5.5	Determine the equation of g in the form $y = a(x + p)^2 + q$.	(3)
5.6	For which value(s) of x will $\dfrac{f(x)}{g(x)} \leq 0$?	(4)
5.7	Use the graphs of f and g to determine the number of real roots of $\dfrac{2}{x} - 5 = -(x-3)^2 - 5$. Give reasons for your answer.	(4)

[20]

QUESTION 6

6.1   Calculate how many years it will take for the value of a truck to decrease to 50% of its original value if depreciation is calculated at 15% per annum using the reducing-balance method.   (4)

6.2   Every month Tshepo deposited R1 500 for his retirement into an account that paid interest at a rate of 9,2% per annum, compounded monthly. Tshepo made his first instalment on his 23rd birthday and the last instalment one month before his 55th birthday. Calculate how much money he had in the account on his 55th birthday.   (5)

6.3   Abram has R150 000 to invest in two separate accounts. One account pays interest at a rate of 8,4% per annum, compounded quarterly, and the other account at a rate of 9,6% per annum, compounded monthly. How much money should he invest in each account so that he will collect the same amount from each account at the end of 12 years?   (6)
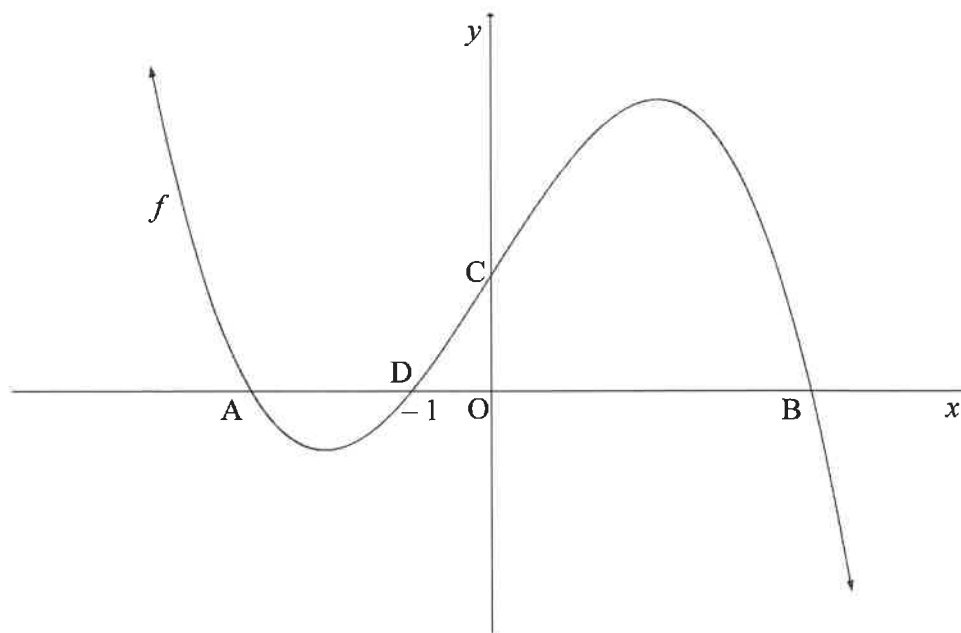   **[15]**

QUESTION 7

7.1   Given: $f(x) = 2 - 3x^2$

        Determine $f'(x)$ from first principles.   (5)

7.2   Determine:

        7.2.1 $D_x\left[(4x+5)^2\right]$ (3)

        7.2.2 $\dfrac{dy}{dx}$ if $y = \sqrt[4]{x} + \dfrac{x^2 - 8}{x^2}$
                                                                                                                              (4)
                                                                                                                              [12]

QUESTION 8

The graph of $f(x) = -x^3 + 13x + 12$ is sketched below.
A, B and D(–1 ; 0) are the x-intercepts of f.
C is the y-intercept of f.

8.1   Write down the coordinates of C.   (1)

8.2   Calculate the coordinates of A and B.   (5)

8.3   Determine the point of inflection of g if it is given that $g(x) = -f(x)$.   (4)

8.4   Calculate the value(s) of x for which the tangent to f is parallel to the line $y = -14x + c$.   (4)

[14]

QUESTION 9

A right circular cone with radius p and height t is machined (cut out) from a solid sphere (with centre C) with a radius of 30 cm, as shown in the sketch.
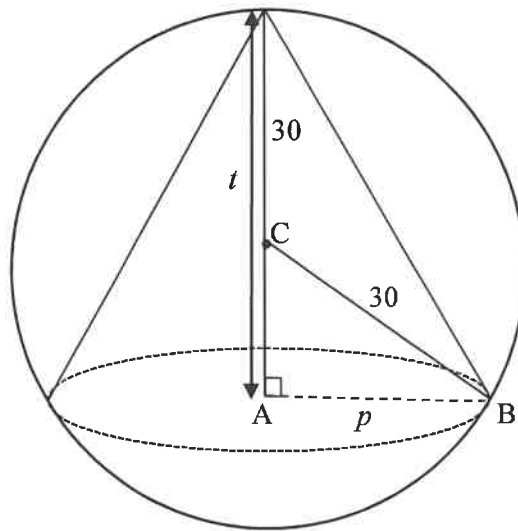
$$\text{Sphere:} \quad V = \frac{4}{3}\pi r^3$$

$$\text{Cone:} \quad V = \frac{1}{3}\pi r^2 h$$

9.1   From the given information, express the following:

  9.1.1   AC in terms of t.   (1)

  9.1.2 p^2, in its simplest form, in terms of t.   (3)

9.2   Show that the volume of the cone can be written as $V(t) = 20\pi t^2 - \frac{1}{3}\pi t^3$.   (1)

9.3   Calculate the value of t for which the volume of the cone will be a maximum.   (3)

9.4   What percentage of the sphere was used to obtain this cone having maximum volume?   (4)

[12]

QUESTION 10

Ben, Nhlanhla, Owen, Derick and 6 other athletes take part in a 100 m race. Each athlete will be allocated a lane in which to run. The athletic track has 10 lanes.

10.1   In how many different ways can all the athletes be allocated a lane?   (2)

10.2   Four athletes taking part in the event insist on being placed in lanes next to each other. In how many different ways can the lanes be allocated to the athletes now?   (3)

10.3   If lanes are randomly allocated to athletes, determine the probability that Ben will be placed in lane 1, Nhlanhla in lane 3, Owen in lane 5 and Derick in lane 7.   (2)

[7]

QUESTION 11

A survey on their preference of exercise was conducted among 140 people in two age groups. The information is summarised below.

AGE	TENNIS	RUNNING	GYM	TOTAL
35 years and younger	a	28	c	80
Older than 35 years	b	21	d	60
	21	49	70	140

11.1   If it is given that preferring to play tennis and age are independent of each other, determine the value of a.   (3)

11.2   If it is given that $a = 12$, determine the probability that a randomly selected person prefers going to the gym or is in the age group 35 years and younger.   (5)

[8]

TOTAL: 150

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad\qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad\qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i} \qquad\qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad\qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \; \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad\qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad\qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad\qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad\qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad\qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad\qquad\qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$